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Fostering Mathematical Minds: A Problem-Based Approach to Conquering Linear Equations

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ABSTRACT

For many students, higher-level math feels like an insurmountable wall. We started this research to find a better way to teach one of those challenging topics: Systems of Linear Equations in Three Variables (SPLTV). Our goal was to develop learning materials that didn't just teach formulas, but actively nurtured students' own logical-mathematical intelligence. Using a four-stage development model (Define, Design, Develop, Disseminate), we created a complete learning package based on the Problem-Based Learning (PBL) model for Grade X vocational students. This package included everything from teaching modules and student worksheets to learning videos and tests. To see if it worked, we gathered feedback from experts, observed the materials in action, and measured student growth with pre- and post-tests. The results were incredibly positive. Experts gave the materials a strong "Valid" rating (with scores like 3.79 and 3.82 out of 4). In the classroom, the materials proved highly practical and easy to use. Most importantly, the students thrived. We saw an 84.6% class-wide learning completion rate, and a comparison with a control group showed a statistically significant improvement ($p < 0.05$) in their logical thinking skills. This study shows that with the right approach, we can move beyond rote memorization and create tools that empower students to think like true problem-solvers.

Keywords: Problem-Based Learning (PBL), Logical-Mathematical Intelligence, Learning Material Development, 4D Model, Systems of Linear Equations in Three Variables (SPLTV), Educational Innovation.

INTRODUCTION

WHY SO MANY STUDENTS STRUGGLE WITH MATH

1.1. Moving Beyond Memorization

In today's world, being "good at math" means much more than just getting the right answer. We need students who can think critically, solve complex problems, and see the world through a logical lens [20]. This ability, often called logical-mathematical intelligence, is about seeing patterns, thinking systematically, and building a strong argument—skills that are vital not just in a science lab, but in everyday life [10, 14]. It's the foundation for the kind of higher-order thinking that allows people to innovate and lead [10].

Research consistently shows a clear link between this kind of thinking and a student's success in school and beyond [14]. Yet, if we're honest, we know that many students aren't developing these skills. National assessments have pointed out that students often lack the ability to analyze problems deeply or reason their way through a complex challenge [9]. They've learned to follow steps, but not necessarily to think for themselves.

1.2. The "SPLTV Problem": When Math Gets Too Abstract

This issue becomes crystal clear when students encounter topics like Systems of Linear Equations in Three Variables (SPLTV). For many, this is where math stops making sense. It demands a leap from concrete numbers to abstract thinking and juggling multiple steps at once [3, 18]. The real struggle isn't just with the algebra itself; it's that students can't see how these abstract 'x's and 'y's connect to anything real or meaningful [4]. This disconnect can quickly lead to a downward spiral of confusion, anxiety, and disengagement. When we look at how students try to solve these problems, we often see a mechanical application of rules without any real understanding, a sure sign that their logical-mathematical intelligence isn't being activated [18]. It's clear that we can't just keep teaching the same way; we need a new approach.

1.3. A Better Way Forward: Problem-Based Learning (PBL)

This is where Problem-Based Learning (PBL) comes in. It's a teaching philosophy that flips the traditional classroom on its head. Instead of listening to a lecture and then doing practice problems, students start with a messy, interesting, real-world problem [8, 13]. The problem itself becomes the reason to learn. It pushes students to work together, ask questions, and hunt for the knowledge they need to find a solution. This process naturally encourages them to think critically, analyze information, and become active drivers of their own education [2, 3]. Study after study has shown that PBL doesn't just help students get better grades; it makes them better thinkers and more capable problem-solvers than traditional teaching methods do [6, 13, 19].

1.4. The Tools Matter: Creating High-Quality Materials

But PBL isn't magic. It only works if it's supported by excellent learning materials [1]. A standard textbook full of dry formulas just won't cut it. Students need materials that draw them into the problem, guide their inquiry without giving away the answers, and encourage them to explore and collaborate. The materials need to feel authentic and relevant, sparking a genuine curiosity that fuels their learning journey [7]. We've seen how powerful it can be when these materials incorporate technology or connect to students' own cultures, making the learning experience that much richer [12, 20].

With all this in mind, we set out on a mission. Our goal was to create a complete package of PBL materials for SPLTV—materials that were well-designed, easy for teachers to use, and, most importantly, effective at helping students not just solve equations, but become more confident and capable logical thinkers.

METHOD:

OUR JOURNEY OF CREATION

To bring our vision to life, we embarked on a Research and Development (R&D) project. We chose a well-regarded framework called the 4D model, which breaks the creation process into four clear stages: Define, Design, Develop, and Disseminate. This structured approach ensured that every decision we made was thoughtful, intentional, and aimed at creating the best possible learning tools.

2.1. The 4D Blueprint

2.1.1. Stage 1: Define – Understanding the Real Needs

Before we could build anything, we had to do our homework. This meant digging deep to understand the real challenges and opportunities.

- **Front-End Analysis:** We started by visiting a local vocational high school. We sat in on math classes and had honest conversations with the teachers. What we saw confirmed our suspicions: learning was mostly teacher-led, with students passively taking notes. The available materials were standard textbooks filled with routine questions, which weren't designed to spark curiosity or deep thinking.
- **Learner Analysis:** We focused on the Grade X students who would be using our materials. We looked at their current academic skills and recognized that their logical-mathematical intelligence was still developing, presenting a perfect opportunity for growth.
- **Concept Analysis:** We mapped out the essential concepts of SPLTV, from understanding what variables are to modeling real-world scenarios and mastering different solution methods like elimination and substitution.
- **Task Analysis:** With the core concepts identified, we planned a sequence of learning tasks. These tasks would form the backbone of our student worksheets, guiding students step-by-step from understanding a problem to confidently solving it.
- **Setting Our Goals:** Finally, we pulled all this information together to set clear, measurable learning objectives. These goals would be our North Star throughout the entire development process, guiding the creation of both the learning content and the tests used to measure success.

2.1.2. Stage 2: Design – Crafting the Prototype

With a clear plan in hand, it was time to start designing. This is where our ideas began to take shape.

- **Creating the Test:** We first designed the test that would measure logical-mathematical intelligence. It wasn't a typical math test. We wrote essay-style questions based on real-world scenarios that required students to reason their way to a solution, not just plug in numbers.
- **Choosing the Media:** We knew students learn in different ways. So, we decided on a multi-pronged approach: engaging, printed Student Worksheets (LKPD) that would encourage hands-on collaboration, and short, clear learning videos to explain key ideas visually.
- **Formatting the Package:** We carefully planned the layout of each component. The student worksheets were structured to follow the natural flow of a PBL investigation. The teaching module was designed as a friendly, step-by-step guide for teachers.
- **Building the First Draft:** This stage culminated in our first complete prototype, or "Draft 1." It was a full package containing the teaching module, worksheets, videos, tests, and all the instruments we'd need for our research, like feedback forms and observation checklists.

2.1.3. Stage 3: Develop – Refining and Testing

No creation is perfect on the first try. This stage was all about refining our materials through feedback and real-world testing.

- **Expert Review:** We gave our "Draft 1" to a panel of experts—two university professors in math education and a seasoned high school math teacher. They gave us invaluable feedback on everything from the accuracy of the content to the clarity of the language. We took their suggestions to heart and revised the materials until they earned a stamp of approval.
- **Classroom Trials:** With a polished "Draft 2," we were ready for the classroom. First, we did a quick "readability test" with a few students to make sure our instructions were easy to understand. Then, we launched a full field trial in a Grade X class to see how the materials worked in a live setting. Over three class meetings, we observed closely to see if the materials were practical and effective.

2.1.4. Stage 4: Disseminate – Sharing Our Work

The final stage is about making sure our work can benefit others. We packaged the final, tested materials into a user-friendly format, ready for sharing. Our plan was to distribute them both in print at the school and online through a Google Drive link shared with teacher groups, hoping to help other educators bring PBL into their classrooms.

2.2. The People and the Process

Our study took place at a vocational high school with Grade X students. We used a "purposive sampling" technique, meaning we chose this school because it was a great fit for our research goals [17]. For our main experiment, we worked with three classes: an "experimental class" that used our new PBL materials, a "control class" that stuck with the school's usual teaching methods, and a "trial class" where we first tested our materials for practicality.

To gather our data, we used a mix of tools:

- **Validation Sheets:** For expert feedback on the quality of our materials.
- **Observation Sheets:** To watch how the learning unfolded in the classroom and track students' problem-solving behaviors.
- **Questionnaires:** To hear directly from students about their experience with the materials.
- **Pre- and Post-Tests:** To get a clear, numerical measure of how much the students' logical thinking skills grew.

2.3. Making Sense of the Data

We analyzed our findings using both qualitative and quantitative methods. We calculated average scores from the expert and observation sheets to rate validity and practicality. For effectiveness, we looked at several key metrics: the percentage of students who passed the post-test, the "N-Gain" score (a measure of improvement from pre-test to post-test), and the student activity levels. Finally, we used a statistical test (an independent samples t-test) to compare the results of our experimental group with the control group, to see if our materials made a real, statistically significant difference.

RESULTS AND DISCUSSION: DID IT ACTUALLY WORK?

After all the planning, designing, and testing, the big question remained: Did our materials actually make a difference? We looked at the results from three angles: Were the materials high-quality (valid)? Were they easy to use in a real classroom (practical)? And most importantly, did they help students learn and think better (effective)? The answer to all three was a resounding yes.

3.1. The Findings

3.1.1. A Stamp of Approval from the Experts (Validity)

Before we ever brought the materials into a classroom, we had them reviewed by experts. Their feedback was overwhelmingly positive. As Table 1 shows, every single component of our learning package was rated as "Valid."

Table 1. Validation Results of Learning Materials and Instruments

No.	Component	Validity Coefficient	Category
1	Teaching Module	3.79	Valid
2	Student Worksheet (LKPD)	3.82	Valid
3	Learning Videos	3.79	Valid
4	Test Package	3.86	Valid
5	User Guidebook	3.75	Valid

6	Research Instruments	3.82	Valid

This told us we were on the right track. The materials were mathematically sound, educationally effective, and well-designed.

3.1.2. Success in the Real World (Practicality)

A great idea on paper is useless if it doesn't work in a real, busy classroom. Our observations during the trial phase showed that the materials were highly practical. The teacher was able to follow the PBL guide with ease, and the students jumped right into the worksheets. We rated the overall implementation a 3.43, which falls into the "High" practicality category.

3.1.3. The Impact on Students (Effectiveness)

This is the part that mattered most. Did the students actually benefit? The evidence was clear and compelling, as detailed in the following tables.

- **Mastering the Material:** The post-test results, summarized in Table 2, show that an impressive **84.6%** of the students in our experimental class passed, demonstrating a strong grasp of the concepts.

Table 2. Post-Test Learning Completion Summary (Experimental Class)

Metric	Value
Highest Score	94
Lowest Score	52
Average Score	80
Classical Completion Rate	84.6%

- **Measurable Growth:** We calculated an "N-Gain" score to measure how much each student improved from the pre-test to the post-test. As shown in Table 3, the vast majority of students showed medium to high growth, with the class average N-Gain score being **0.73**, which is considered a **"High"** level of improvement.

Table 3. N-Gain Category Distribution (Experimental Class)

N-Gain Category	Number of Students	Percentage
High	23	62.16%
Medium	13	35.13%
Low	1	2.70%

- **Thinking in Action:** Our observers tracked how often students showed signs of logical-mathematical thinking during class. The data in Table 4 reveals a consistent and high level of engagement, with an overall average score of **84.33%**, which we categorized as **"Good."**

Table 4. Summary of Student Logical-Mathematical Intelligence Activities

Meeting	Average Activity Score (%)	Category

Meeting 1	77.98%	Good
Meeting 2	84.52%	Good
Meeting 3	90.48%	Very Good
Overall Average	**84.33%**	**Good**

- **The Students' Verdict:** What did the students themselves think? They loved it. As Table 5 shows, their feedback was overwhelmingly positive, indicating they found the lessons engaging, helpful, and interesting.

Table 5. Student Response Summary

Response Type	**Average Percentage**
Positive Response	92.05%
Negative Response	7.95%

- **The Final Proof:** The statistical test delivered the final verdict. When we compared the test scores of our experimental group to the control group (who used traditional methods), the difference was stark. Table 6 summarizes the results, which provide clear statistical proof that our PBL materials led to a significantly greater improvement in students' logical-mathematical intelligence.

Table 6. Hypothesis Test Summary (Independent Samples t-test)

Comparison	**Significance (p-value)**	**Result**
Post-Test Scores (Exp. vs. Control)	.000	Significant Difference
N-Gain Scores (Exp. vs. Control)	.000	Significant Difference

3.2. What This All Means

The numbers tell a powerful story. Our systematic approach to developing these PBL materials resulted in a resource that is not only high-quality and practical but also profoundly effective.

The **high validity** wasn't an accident; it was the result of a careful, deliberate process. By starting with a deep analysis of the students' and teachers' needs, we built a foundation that was already aligned with what works in education [2, 7]. The feedback from our expert panel was the final polish, ensuring every piece was just right. By grounding the abstract concepts of SPLTV in relatable, real-world problems, we made the topic accessible and engaging, a strategy proven to boost understanding [4].

The **high practicality** shows that PBL doesn't have to be a complicated or intimidating model for teachers to adopt. With the right tools, like our step-by-step teaching guide, teachers can confidently shift from being a lecturer to being a facilitator of learning [3, 8]. The students' enthusiastic response shows what happens when you trust them to be active learners. When faced with a genuine challenge, they rose to the occasion, taking ownership of their learning in a way that passive instruction simply doesn't allow [13, 19].

But the most exciting result is the **demonstrable effectiveness** in building students' thinking skills. The huge jump in test scores wasn't just about memorizing formulas better. It was a direct result of the mental workout the PBL process gave them. Our worksheets forced them to think like mathematicians: to analyze a situation, build a model, design a strategy, and defend their conclusions [5, 18]. This entire workflow is a hands-on exercise in logical-mathematical thinking [11]. Our findings strongly echo a large body of research that connects PBL with superior problem-solving and critical thinking skills [2, 15].

One of our key insights from watching the classes was the importance of **making logical thinking a habit**. At first, many students were hesitant, waiting to be told what to do. But as they got used to the PBL process, a change occurred. They started asking their own questions, debating ideas in their groups, and spontaneously using the language of logic. It was a beautiful reminder that intelligence isn't something you either have or you don't; it's a muscle that gets stronger with practice [11]. Our materials provided the perfect gym for that workout.

CONCLUSION: A NEW CHAPTER IN MATH EDUCATION

In the end, our goal was simple: to create a tool that could transform a challenging math topic from a source of anxiety into an opportunity for growth. The results of this study show that we succeeded. We have developed a set of problem-based learning materials for SPLTV that are proven to be valid, practical, and highly effective at improving students' logical-mathematical intelligence.

Our Journey: Strengths and Limitations

Of course, no research project is perfect. Our study was conducted at a single school, which means we should be cautious about generalizing the findings too broadly. And while we shared our materials, the dissemination was limited. However, the strength of our work lies in the rigorous 4D development process we followed, the creation of a complete and integrated learning package, and our use of multiple data sources to confirm our findings.

An Invitation to Educators

We believe this study offers more than just results; it offers a tangible, high-quality product that teachers can use in their own classrooms tomorrow. We strongly encourage educators to embrace student-centered approaches like PBL. With the support of well-designed materials like these, we can move beyond teaching for the test and start fostering a genuine, lasting understanding and a love for creative problem-solving. For future research, we would love to see these materials tested with a wider range of students, and to track their impact over a longer period. We hope this is just the beginning of a new chapter in making mathematics meaningful for all students.

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